SEPTEMBER 19, 2017
CLASS NOTES.

LAST CLASS

lemma 1.60: if a language is regular, then it has a RE

lemma 1.55: if a language L has a RE, then it is regular

Pf/ let R be a RE for L

construct an NFA N recognizing L

consider the 6 cases from def'n of RE's:

① $R = a$ (single symbol) for some $a \in \Sigma$ $\longmapsto$ $\rightarrow \bigcirc \xrightarrow{a} \circledcirc$

② $R = \emptyset$ (empty set) $\longmapsto$ $\rightarrow \bigcirc$

③ $R = \varepsilon$ (empty string) $\longmapsto$ $\rightarrow \circledcirc$

④ $R = R_1 \cup R_2$ (union) $\longmapsto$

⑤ $R = R_1 \circ R_2$ (concatenation) $\longmapsto$

⑥ $R = R_1^*$ (star) $\longmapsto$

use constructions from class for taking union, concatenation, or star

■

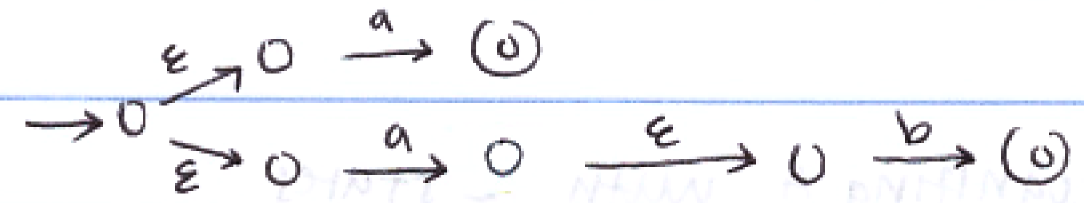
example $R = (ab \cup a)^*$ $\Sigma = \{a, b\}$

① a $\rightarrow \bigcirc \xrightarrow{a} \circledcirc$

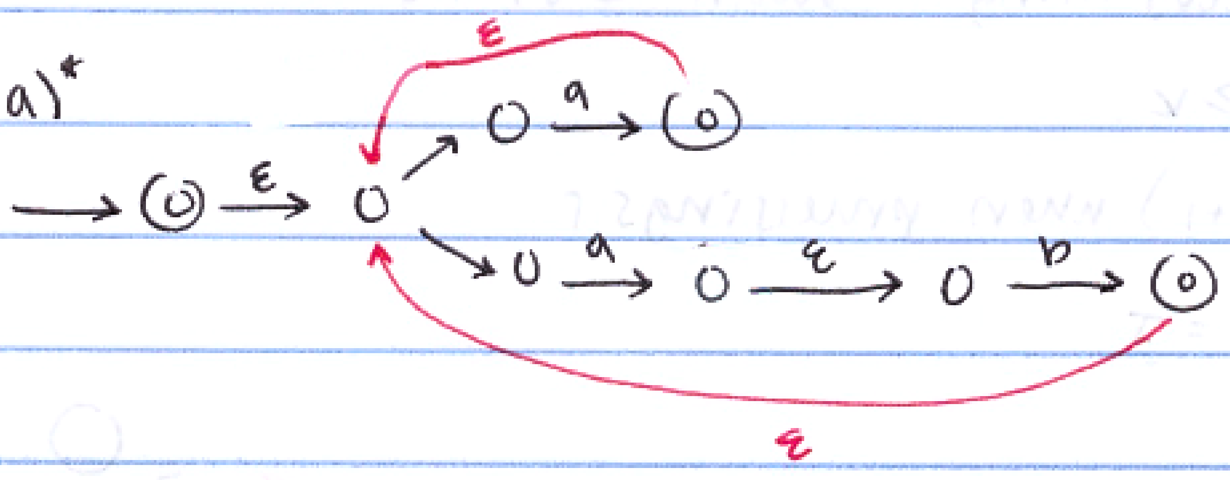
② b $\rightarrow \bigcirc \xrightarrow{b} \circledcirc$

③ ab (a∘b) $\rightarrow \bigcirc \xrightarrow{a} \bigcirc \xrightarrow{\varepsilon} \bigcirc \xrightarrow{b} \circledcirc$

④ ab ∪ a



⑤ (ab ∪ a)*



regular
DFA = NFA = RE

?

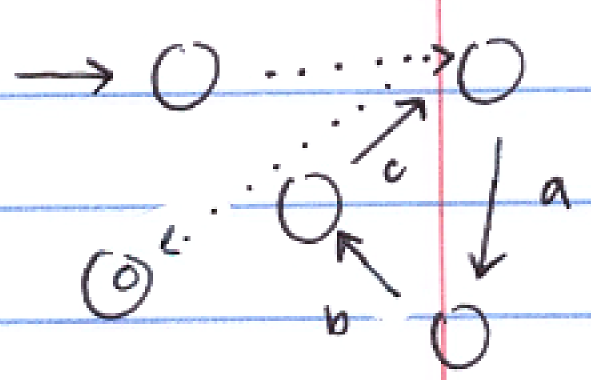
FACT: Every skunk has a white stripe
$\Rightarrow$ NO STRIPE = NOT A SKUNK!

FACT every regular language has some property P
$\Rightarrow$ if language L lacks property P, L is not regular

contrapositive
$A \Rightarrow B$
$!B \Rightarrow !A$

what is property P?
observation: let M be a DFA with k states
let $x \in \Sigma^*$ be an input of length $|x| \geq k$

conclusion

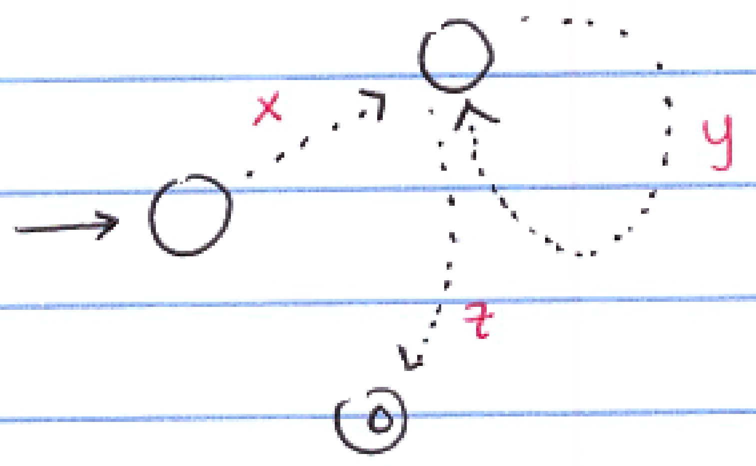


$\exists$ state $q \in Q$ visited twice when running M on x (pigeonhole principle)

conclusion Suppose x is an accepted $ $x = \underbrace{\ldots}_{x}\underbrace{ab}_{y}\underbrace{c\ldots}_{z}$
Then $x = \underbrace{\ldots}_{x}\underbrace{(abc)^*}_{y^*}\underbrace{\ldots}_{z}$ is also accepted

Pumping Lemma (Thm 1.70)
Let A be a regular language. There exists a "pumping length" $p > 0$, such that
$\forall \boxed{s \in A}$ of length $|s| \geq p$, there exists $x, y, z \in \Sigma^*$ s.t. $s = xyz$ and:
s is accepted by A
① $\forall i \geq 0 \quad xy^iz \in A$
② $|y| > 0$
③ $|xy| \leq p$

Pf/ Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA recognizing A with k states
Let $s = s_1 s_2 \ldots s_n \in A$ s.t. $|s| = n \geq k$.
Suppose M enters states $(r_1, r_2, \ldots r_{n+1})$ when processing s
By pigeonhole principle $\exists\ i < j$ s.t. $r_i = r_j$
Pick smallest such j
Note $j \leq k+1$, since M has only k states
$\therefore$ Let $x = s_1 s_2 \cdots s_{i-1}$; $y = s_i \cdots s_{j-1}$; $z = s_j \cdots s_n$



Let's verify the 3 conditions

let $p = k$

① since x takes M from r_1 to r_i,

y takes M from r_i to r_j,

z takes M from r_j to r_{n+1},

hence M accepts xy^iz $\forall i \geq 0$

② since $i \neq j$, $|y| > 0$

③ since $j \leq k+1$, $|xy| \leq k = p$ ■

example let $B = \{0^n 1^n \mid n \geq 0$. Claim $\Rightarrow$ B NOT regular

pf/ via contradiction. Assume B is regular

$\therefore$ $\exists$ pumping length $p > 0$.

Pick $s = 0^p 1^p$. $\Rightarrow$ preconditions (a) $s \in B$ ✓

(b) $|s| = 2p \geq p$ ✓

$\therefore$ $\exists x, y, z$ s.t. $s = xyz$, and:

by prop ③ $|xy| \leq p$, x & y contain only 0's

By prop ② $|y| > 0$, y contains at least one 0

By prop ① $xy^2z = xyyz$ has more zeroes than ones, ie. $xy^2z = 0^{p'} 1^p$

for $p' > p$ $\therefore$ $xy^2z \notin B$

CONTRADICTION! ■